ejercicio 5: variable aleatoria discreta

1. Binomial.

C = cured
$\bar{C}$ = not cured

$P(x=k) = \binom{n}{k} \cdot p^k (1-p)^{n-k}$

$P(C) = 0,85$
$P(\bar{C}) = 0,15$

$B(n;p) = B(3;0,85)$

a) $P(x=3) = \binom{6}{3} \cdot 0,85^3 \cdot 0,15^{\overset{6-3}{3}} = 0,0424$ //

b) at least 4 of them cured = 4 or more = $x \geq 4$

0 0 0 0 0 0
1 2 3 4 5 6

$P(x \geq 4) = P(x=4) + P(x=5) + P(x=6) =$

$= \underbrace{\binom{6}{4}}_{15} \cdot 0,85^4 \cdot 0,15^{6-4} + \underbrace{\binom{6}{5}}_{6} \cdot 0,85^5 \cdot 0,15^{6-5} + \underbrace{\binom{6}{6}}_{1} \cdot 0,85^6 \cdot \cancel{0,15^0} =$

$= 0,1762 + 0,3993 + 0,377 = 0,9525$ //

2. Law of rare events: poisson

states that bacteria/1mm³ → which is a density so ⟹ $P(\lambda = np)$

There fore, what is "n" and "p"?
↳trials ↳success prob.

$p = 0,002$ $n = 1cm^3 = 1000mm^3$

n = nº of independent trials/ opportunities to get bacteria.

Probability of 5 bacteria at most =

= 5 bacteria or less = $P(x \leq 5)$
(no more)

x = nº of success / bacteria that appear.

$\lambda = np = 1000 \cdot 0,002 = 2$

$P(x) = e^{-\lambda} \cdot \frac{\lambda^x}{x!}$

$P(0) = e^{-2} \cdot \frac{2^0}{0!} = e^{-2}$
$P(1) = e^{-2} \ldots$
$P(2) = \ldots$
$P(3) = \ldots$

$P(x \leq 5) = e^{-2} \left(\frac{\lambda^x}{x!} \right) =$

↖ P(0) + P(1) + P(2)...

faster way
saco factor común de e^{-2} y ya.

$= e^{-2} \left(\frac{2^0}{0!} + \frac{2^1}{1!} + \frac{2^2}{2!} + \frac{2^3}{3!} + \frac{2^4}{4!} + \frac{2^5}{5!} \right) = e^{-2} \cdot 7,16 = 0,969$ //

$= e^{-2} (1 + 2 + 2 + 1,3 + 0,6 + 0,26)$
↑ P(0) ↑ P(1)

3. poisson

120 calls/min

a) is asking calls per 2 seconds. Since we have minutes, we turn them into secs and then adjust for 2 seconds.

$$\frac{120 \text{ calls}}{\cancel{\text{min}}} \cdot \frac{1 \cancel{\text{min}}}{60 s} = \frac{120 \text{ calls}}{60 \ s}$$

To 2seconds:

60 seconds = 2second · x ⇒ 30seconds.

Divide by 30 each.

$$\left(\frac{120}{60}\right)\begin{matrix}:30\\:30\end{matrix} = 4 \text{calls}/2 \text{seconds} \Rightarrow \lambda = 4 \text{ calls}/2 \text{ seconds}$$

a) $P(x < 4) = P(0) + P(1) + P(2) + P(3)$

$$= e^{-4}\left(\frac{4^0}{0!} + \frac{4^1}{1!} + \frac{4^2}{2!} + \frac{4^3}{3!}\right) = 0{,}4335_{\shortparallel}$$

factor común (easier)

b) change units agains, we need three seconds, not 2. ⇒ $3 = 2 \cdot x$

$x = 1{,}5$

so multiply each side by 1,5

1. $\left(4 \text{ calls}/2 \text{ seconds}\right)\begin{matrix}\times 1{,}5\\ \times 1{,}5\end{matrix} = 6 \text{ calls}/3 \text{ seconds}$

or

2. $\left(2 \text{ calls}/\text{sec}\right)\begin{matrix}\times 3\\ \times 3\end{matrix} = 6 \text{ calls}/3 \text{seconds}$

$P(x \geq 3) = P(3) + P(4) \dots$ P(a lot) ⇒ instead ⇒

"at least" → 3 or more $= (x \geq 3)$

⇒ $P(x \geq 3) = 1 - P(2) - P(1) - P(0) = 1 - \left[P(2) + P(1) + P(0)\right] =$

$$= 1 - \left[e^{-6}\left(\frac{6^2}{2!} + \frac{6^1}{1!} + \frac{6^0}{0!}\right)\right] = 1 - \left[e^{-6}\left(18 + 6 + 1\right)\right] =$$

$$= 1 - \left[e^{-6}(25)\right] = 1 - 0{,}0619 = 0{,}938_{\shortparallel}$$

4. Test + binomial

⇒ Diagnostic test

D = disease
$\bar{D}$ = not disease

P = positive
N = negative

$P(+) = 0{,}01 = 1\%$

	D	$\bar{D}$	total
Test +	true positive (TP)	false + (FP)	0,01
Test −	true negative (TN)	false ne. (FN)	0, 99

negative predictedted values (NPV) = 0,98 → $P(\bar{D}/-)$

positive " " (PPV) = 0,95 → if +, what's the chance the person actually has the disease. $= P(D/+)$

a)

$P(D) = ?$

1st ⇒ what do we know?

so... despejamos

$P(\bar{D}/-) = \frac{P(\bar{D}\cap -)}{P(-)} = 0{,}98$; $P(\bar{D}\cap -) = 0{,}98 \cdot P(-) = 0{,}98 \cdot 0{,}99 = 0{,}9702$

$P(D/+) = \frac{P(D\cap +)}{P(+)} = 0{,}95$; so... $P(D\cap +) = 0{,}95 \cdot P(+) = 0{,}95 \cdot 0{,}1 = 0{,}0095$

$1\% = 0{,}01$

Rellenamos la tabla

	D	$\bar{D}$	total
Test +	0,0095	0,0005	0,01
Test −	0,0198	0,9702	0,99

Calculate para $\bar{D}$

$P(+\cap\bar{D}) = P(+) - P(+\cap D) = 0{,}01 - 0{,}0095 = 0{,}0005$

$P(-\cap D) = P(-) - P(-\cap\bar{D}) = 0{,}99 - 0{,}9702 = 0{,}0198$

despejado de ⇒ $P(-) = P(-\cap\bar{D}) + P(-\cap D)$

So the prevalence is

a) $P(D) = P(D\cap +) + P(D\cap -) = 0{,}0095 + 0{,}0198 = 0{,}0293$

b) sensitivity is by definition $P(D/+)$, already calculated.

summery: PPV $= P(D/+) = 0{,}95$

0,95

$P(D/+) = \boxed{\frac{P(D\cap +)}{P(+)}} = 0{,}95$ → $P(D\cap +) = 0{,}95 \cdot P(+) = 0{,}0095$

0,01 (data of problem)

specifity is by def.: $P(\bar{D}/-)$

summery: NPV $= P(\bar{D}/-) = \frac{P(\bar{D}\cap -)}{P(-)}$ ⇒ $P(\bar{D}\cap -) = 0{,}98 \cdot P(-) = 0{,}9702$

0,98

$1 - 0{,}1$

$1 - P(+)$

c) $B(n, p)$; $B(12, P(\text{wrong diagnosis})) = B(12; 0{,}0203)$

$P(\text{wrong diagnosis}) = P(+\cap\bar{D}) + P(-\cap D) = 0{,}0005 + 0{,}0198 = 0{,}0203$

$P(x \geq 1)$

WRONG!
the 12 people are sick
so a wrong diagnosis is just $P(D\cap -)$.

c) correction

$n = 12$ sick people

$p = P(-/D)$ not $P(-\cap D)$ because we know they are sick, and we are picking a negative from the group of sick not a negative from total population.

so... $P(-/D) =$

$$= \frac{P(-\cap D)}{P(D)} = \frac{0{,}0198}{0{,}0095 + 0{,}0198} = \frac{0{,}0198}{0{,}0293} = 0{,}676$$

$B(12;\ 0{,}676)$

$$P(x \geq 1) = 1 - [P(x=0)] = 1 - \binom{12}{0} \cdot 0{,}676^0 \cdot (1-0{,}676)^{12-0} =$$

$0 \mid 1 \ \ 2 \ \ 3 \ \cdots \infty$

$1 - P(x=0)$

$$= 1 - (1 \cdot 1 \cdot (0{,}324)^{12}) = 0{,}999 \simeq 1$$

d) $n = 12$ people (random/general)

$p = P(\text{Right diagnosis}) = P(D\cap +) + P(\bar{D} \cap -) = 0{,}0095 + 0{,}9702 = 0{,}9797$

$$P(x=12) = \binom{12}{12} \cdot 0{,}9797^{12} \cdot \cancel{(1-0{,}9797)}^{\,12-12=0 \to 1} = 1 \cdot 0{,}9797^{12} = 0{,}7818$$

5. rare events + poisson + Binomial (exam type)

Turner $= T$ $\qquad P(T) = 1/2000 = 5 \cdot 10^{-4}$

narrowing aorta $= a$ $\qquad P(a/T) = \frac{1}{10} = 0{,}1$

a) $n = 4000$

$p = P(T) = 5 \cdot 10^{-4}$ → since probability is very small in a large $n \Rightarrow$ poisson not binomial

$\Rightarrow$ error calculator

$\lambda = np = 4000 \cdot 5 \cdot 10^{-4} = 2$; $\lambda = 2$ $\qquad f(x) = e^{-\lambda} \cdot \frac{\lambda^x}{x!}$

1. $\lambda(x>3) = 1 - P(3) - P(2) - P(1) - P(0) = 1 - [P(0) + P(1) + P(2) + P(3)] =$

$$= 1 - \left[e^{-2}\left(\frac{2^0}{0!} + \frac{2^1}{1!} + \frac{2^2}{2!} + \frac{2^3}{3!}\right)\right] = 1 - [e^{-2}(1 + 2 + 2 + 1{,}33)] =$$

$$= 1 - 0{,}8571 = 0{,}1429$$

2. "some woman" → any from the sample $\Rightarrow P(a \cap T)$; $P(a/T) = \frac{P(a\cap T)}{P(T)} \Rightarrow$

$\Rightarrow P(A\cap T) = P(T) \cdot P(a/T) =$

$= 5 \cdot 10^{-4} \cdot 0{,}1 = 5 \cdot 10^{-5}$

$n = 4000$; $p = 5 \cdot 10^{-5}$; $\lambda = np = 0{,}2$

"at least" = some women $\Rightarrow x \geq 1$

$$P(x \geq 1) = 1 - P(0) = 1 - \left(e^{-0{,}2} \cdot \frac{0{,}2^0}{0!}\right) = 0{,}181$$

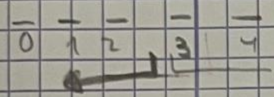
b) next page $\Rightarrow$

exercise 5.

5. rare ...

b) $n = 20$ $\quad p = P(a/T) = 0,1$

here we can use binomial because probability is small but so is our n. ☺



$$P(X<3) = P(X=0) + P(X=1) + P(X=2) =$$

$$= \binom{20}{0}(0,1)^0(0,9)^{20} + \binom{20}{1}(0,1)^1(0,9)^{19} + \binom{20}{2}(0,1)^2(0,9)^{18} =$$

$$= 1 \cdot 1 \cdot 0,9^{20} + 20 \cdot 0,1 \cdot 0,9^{19} + 190 \cdot 0,01 \cdot 0,9^{18} = 0,6766$$